Active Estimation of Mass Properties for Safe Cooperative Lifting

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***Abstract*— This work considers estimation of mass parameters for multi-robot coordinated lifting in the context of coordinated aerial manipulation, and develops strategies for active parameter estimation for cooperative manipulation tasks through an information-theoretic framework. The active sensing problem is formulated based on application of increasing forces to the object and detection of small motions that occur when the center of pressure exits the convex hull formed by existing contacts. In order to enable identification of informative actions, we develop and employ a closed-form solution of Cauchy-Schwarz quadratic mutual information (I_{CS}) for non-parametric filters. The evaluation considers iterative selection from a finite set of measurements and demonstrates that choosing measurements to maximize I_{CS} significantly improves the convergence rate of the parameter estimates compared to random and cyclic selection methods. This approach is extended to consider actuator constraints and feasible lifting configurations and achieves an 80% success rate in formation of feasible lifting configurations compared to a 53% baseline performance.**

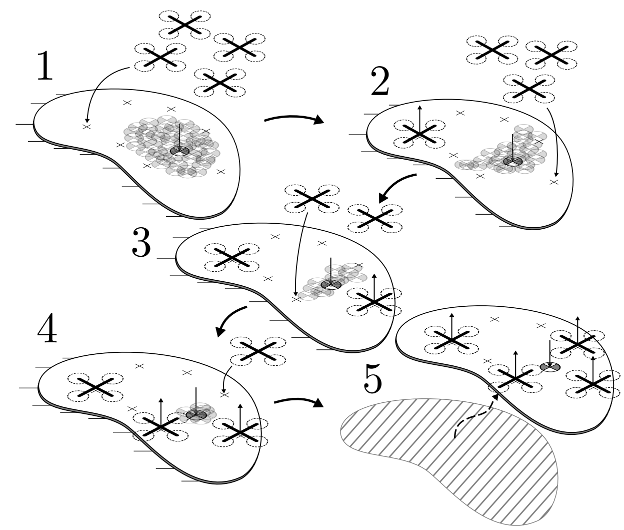

Fig. 1: The sequence of approach, attachment, estimation, and liftoff. (1) Robots approach an object with unknown mass and location of center of mass (CoM). (2) A robot attaches to the object and applies a force in order to reduce uncertainty in the estimate of the mass and CoM. (3) A second robot attaches and applies a force that further reduces uncertainty. (4) A third robot attaches, and two robots apply forces cooperatively. (5) The robots lift the object after estimating the mass properties and forming a feasible lifting configuration.

I. Introduction

Aerial manipulation with micro-air vehicles enables transportation and delivery of payloads with mass properties that may exceed the capacity of individual systems while benefiting from the maneuverability and accessibility of small-scale aerial platforms. However, in scenarios such as search and rescue where object mass properties are unknown the team must concurrently estimate mass properties and engage with the object toward enabling a cooperative object lift.

This work addresses scenarios in which a multi-robot team must lift an object with unknown mass and center of mass (CoM) with hidden cavities or unknown composition such as a piece of rubble. Existing work on aerial manipulation typically assumes known mass properties and foreknowledge of a configuration of robots capable of lifting the object [1, 2]. We relax these assumptions and propose an approach that enables estimation of mass properties and formation of feasible lifting configurations.

We propose an iterative approach for simultaneous parameter estimation and deployment of a multi-robot team equipped with sufficient sensing to inspect an object in order to extract *contact constraints* and viable *attachment points* before beginning the estimation task. As large object motions may lead to unsafe conditions, robots apply increasing force until detecting the initiation of object motion. The correspondence between the applied wrench and initiation of motion is used to infer the object mass and CoM via a non-parametric Bayesian filter. This approach is applicable to a variety of existing aerial manipulation systems such as those based on rigid attachment [3], cable-suspension [4], robotic arms [5], and the analogous multi-robot systems [1, 2, 6] and only requires that robots be able to apply controlled wrenches to the object. The proposed approach seeks to extend these aerial manipulation systems via cooperative estimation and construction of feasible lifting configurations.

Payload state [7] and parameter [3, 8, 9] estimation techniques proposed in the context of aerial manipulation assume already airborne vehicles and formulate traditional or adaptive feedback controllers that seek to regulate the payload state given the known or estimated object model and feasibility assumptions. The approach described in this work seeks to enable more broad application of these methods by constructing configurations of robots capable of lifting target objects and providing parameter estimates that facilitate initialization of these controllers, ensuring that any assumptions

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are met.

In order to quickly achieve accurate estimates, we pursue active selection of informative actions [10] generated by physical interactions [11] with the object. Actions are selected to maximize mutual information between measurements and the target parameters and are used to update a non-parametric Bayesian filter. Similar problem forms are considered in the robotics literature in the context of target localization by mobile robots [12, 13] often estimating target location via particle filtering. We employ Cauchy-Schwarz quadratic mutual information to select from finite sets of actions and provide a closed-form expression for non-parametric filters generalizing the formulation for ranging sensors by Charrow et al. [14].

We approach the problem of parameter estimation and formation of feasible lifting configurations through a series of three scenarios that iteratively introduce components of the full problem:

- *Pure estimation*: iterative selection from a set of wrench measurements or attachment points seeking accurate estimates per number of iterations [15].
- *Estimation and deployment*: robots iteratively attach to the object and apply wrenches individually or collectively given actuator limits that require cooperation to estimate properties of heavy loads.
- *Feasible lifting*: robots simultaneously estimate mass properties and form lifting configurations, as illustrated in Fig. 1, to enable aerial manipulation via a sequence of discovery, inspection, lifting, refinement, and execution of intended manipulation.

Simulations of the estimation task demonstrate rapid convergence of parameter estimates for the active sensing approach. This is integrated into approaches for deployment, lifting, and subsequently multi-robot aerial manipulation of unknown objects, demonstrating significantly increased success rates in formation of feasible lifting configurations compared to default application of the best individual configuration.

II. Approach

This section presents the approach to lifting a payload, estimation of mass properties, and active measurement selection and begins with investigation of feasible lifting configurations and their existence (Sect. II-A). Unknown mass properties are estimated via a wrench-based measurement model and Bayesian filtering (Sects. II-B–II-C). Sections II-D–II-E detail action selection for reduction of uncertainty in mass property estimates and formation of feasible lifting configurations.

Consider a team of n_r robots and an object with unknown mass properties as illustrated in Fig. 2. Define the known and finite set of attachment points $\mathcal{A}$, occupied attachment points $\mathcal{R} \subseteq \mathcal{A}$ with number attached $n_a = |\mathcal{R}|$, and set of remaining attachment points $\hat{\mathcal{A}} = \mathcal{A} \setminus \mathcal{R}$. The object has CoM location $\mathbf{q}_g$, mass m, and resulting gravity wrench $\mathbf{w}_g = -mg \begin{bmatrix} \mathbf{e}_3{}^{\mathrm{T}} & (\mathbf{q}_g \times \mathbf{e}_3)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$. The object is subject to

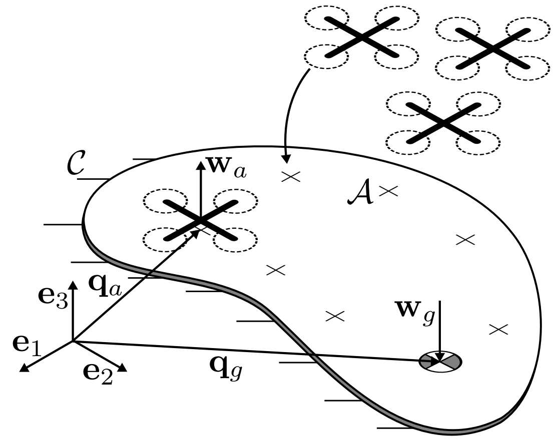


Fig. 2: A team of robots estimate the gravity wrench $\mathbf{w}_g$ of an object with CoM $\mathbf{q}_g$ and contacts $\mathcal{C}$. Robots select attachment points $a \in \mathcal{A}$ at locations $\mathbf{q}_a$. A robot applies a wrench $\mathbf{w}_a$ to the object and infers the location of the CoM by detecting the initiation of object motion.

a set of planar contacts $\mathcal{C}$ with normals in $\mathbf{e}_3$. The contact locations are $\mathbf{q}_c$ for all $c \in \mathcal{C}$ resulting in force-normalized wrenches $\mathbf{w}_c = \begin{bmatrix} \mathbf{e}_3{}^{\mathrm{T}} & (\mathbf{q}_c \times \mathbf{e}_3)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$. This condition ensures no friction forces when applying forces in $\mathbf{e}_3$ although contacts may nominally be subject to friction. Given an applied wrench $\mathbf{w}$, the static equilibrium condition is

$$\mathbf{W}_{\mathcal{C}} \boldsymbol{\lambda}_{\mathcal{C}} + \mathbf{w}_g + \mathbf{w} = \mathbf{0}, \quad \boldsymbol{\lambda}_{\mathcal{C}} \geq 0 \tag{1}$$

where $\boldsymbol{\lambda}_{\mathcal{C}}$ is the vector of reaction forces and $\mathbf{W}_{\mathcal{C}}$ is the contact wrench matrix with columns $\mathbf{w}_c$.

A. Feasible lifting configurations

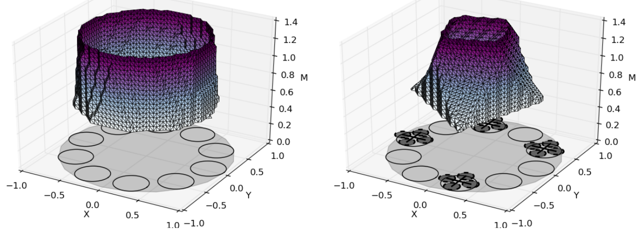
In order to compute the feasibility of lifting the object given a configuration of attached robots $\mathcal{R}$, the feasibility condition is

$$\mathbf{W}_{\mathcal{R}} \boldsymbol{\lambda}_{\mathcal{R}} + \mathbf{w}_g = 0, \quad 0 \leq \boldsymbol{\lambda}_{\mathcal{R}} \leq f_{\max} \tag{2}$$

with applied forces $\boldsymbol{\lambda}_{\mathcal{R}}$, normalized applied wrenches $\mathbf{W}_{\mathcal{R}}$, and robot actuator limit $f_{\max}$. Equation (2) can be extended to determine the existence of feasible configurations given attached and remaining robots via a boolean vector $\mathbf{b} \in \{0, 1\}^{|\hat{\mathcal{A}}|}$ representing the attachment of additional robots and a constraint based on the number of remaining robots to form a mixed-integer linear program:

$$\begin{aligned} &\mathbf{W}_{\mathcal{R}} \boldsymbol{\lambda}_{\mathcal{R}} + \mathbf{W}_{\hat{\mathcal{A}}} \boldsymbol{\lambda}_{\hat{\mathcal{A}}} + \mathbf{w}_g = 0 \\ &0 \leq \boldsymbol{\lambda}_{\mathcal{R}} \leq f_{\max}, \quad 0 \leq \boldsymbol{\lambda}_{\hat{\mathcal{A}}} \leq f_{\max} \mathbf{b} \\ &\sum_{b \in \mathbf{b}} b \leq n_r - n_a. \end{aligned} \tag{3}$$

Sets of mass and CoM values for which feasible lifting configurations exist by (3) are presented in Fig. 3 for $n_a = 0$ and $n_a = n_r$ attached robots in a lifting scenario that will be detailed in later discussion. With no robots attached, feasible configurations can be constructed for parameters covering 79% of the support. When $n_a = n_r$, (2) and (3)



(a) Initial feasibility **(b)** Four robots

Fig. 3: The set of mass and CoM values for which a feasible configuration can be obtained for $n_r = 4$ as computed by (3) using the parameters of the *feasible lifting* scenario in Sect. III. (a) shows the maximal feasible set given no robots attached with a feasibility rate of 79.0% and (b) the feasible set for the best complete deployment of the alloted four robots with a 37.7% feasibility rate, found by exhaustive search.

are equivalent and lifting is feasible for only 37.7% of the support.

B. Wrench-space measurements

Consider the ray in wrench-space $\mathbf{w} = \mathbf{w}_0 + \mathbf{w}_r \lambda_r$ where $\lambda_r \geq 0$. Substituting into (1), the *critical force* f_c is

$$\begin{aligned} f_c = &\arg\max_{\lambda_r} \ \lambda_r \\ \text{s.t. } &\mathbf{W}_{\mathcal{C}} \boldsymbol{\lambda}_{\mathcal{C}} + \mathbf{w}_g + \mathbf{w}_0 + \mathbf{w}_r \lambda_r = 0 \\ &\lambda_r \geq 0, \ \boldsymbol{\lambda}_{\mathcal{C}} \geq 0. \end{aligned} \tag{4}$$

after which the object ceases to be statically stable (1) and begins to move. The object is assumed to return to the original resting condition after a sufficiently small disturbance, and the robots are assumed to be able to detect sufficiently small motions. Measurements of f_c are then incorporated to safely estimate the mass parameters.

Robots detect applied forces through either a calibrated thrust model or other instrumentation. Measurements of critical forces are modeled as Gaussian and are sensitive to uncertainty in the timing and detection of motion and the application and measurement of forces. For a measurement $\mathcal{Z}$ of f_c with realization $z \in \mathcal{Z}$, the measurement distribution is

$$\begin{aligned} p(\mathcal{Z} = z | \mathbf{w}_g, \mathbf{W}_{\mathcal{C}}) &= \mathcal{N}(z - \hat{f}_c, \sigma^2) \\ \hat{f}_c &= \begin{cases} f_c & f_c < f_{\text{meas}} \\ f_{\text{meas}} & f_c \geq f_{\text{meas}} \end{cases} \end{aligned} \tag{5}$$

where $\mathcal{N}(x, \sigma^2)$ is the zero-mean normal pdf with variance σ^2 evaluated at x. The measurement is saturated to obtain $\hat{f}_c$ based on the maximum applied force f_{meas} which will vary from the actuator limit ($f_{\max}$) for cooperative measurements.

We consider measurements with applied forces in the $\mathbf{e}_3$ direction. For a measurement taken by a single robot at attachment point $a \in \mathcal{A}$, the measurement is defined by associated wrench $\mathbf{w}_r = \mathbf{w}_a$. For multiple robots, consider cooperative measurements by subsets of robots contributing in equal proportions. Let $\mathscr{C}_n(X) = \{C | C \subseteq X, 0 < |C| \leq n\}$ be the set of non-empty subsets of some set, X of size at most n. A cooperative measurement $\mathcal{Z}_{\mathcal{M}}$ for some $\mathcal{M} \in \mathscr{C}_{n_r}(\mathcal{R})$ has associated normalized wrench $\frac{1}{|\mathcal{M}|} \sum_{a \in \mathcal{M}} \mathbf{w}_a$ and maximum force $f_{\text{meas}} = |\mathcal{M}| f_{\max}$.

C. Representation of the belief

Given the measurement model, the mass parameters can be estimated using Bayesian filtering. The filtering variable $\theta = \{\mathbf{q}_{g1}, \mathbf{q}_{g2}, m\}$ is defined in terms of mass and the horizontal location of the CoM (where $\mathbf{q}_{gi}$ is the i^{th} element of $\mathbf{q}_g$) and has support Θ with $\theta \in \Theta$, covering the convex hull of the contact points (for which the object is stable) and a range of masses. Belief distributions arising in this problem are often multi-modal or banana-shaped due to the non-linearity of the measurements and motivate use of a non-parametric filter [16]. A histogram filter is used with discretization $\hat{\Theta} \subset \Theta$ according to a regular grid and cell volume δ with a uniform prior over mass and CoM.

D. Information-theoretic measurement evaluation

As is common in active sensing, we select informative measurements to maximize the mutual information (MI) with the target variable. We employ Cauchy-Schwarz quadratic MI [14, 17] as it is readily computed in closed-form which is derived from the Cauchy-Schwarz divergence

$$\begin{aligned} &D_{CS}(f, g) = \\ &\quad \log \int f(x)^2 dx + \log \int g(x)^2 dx - 2 \log \int f(x) g(x) dx. \end{aligned} \tag{6}$$

Cauchy-Schwarz quadratic MI is the divergence between the joint $p_{X_{12}}$ and the product of marginals $p_{X_1} p_{X_2}$:

$$I_{CS}(X_1; X_2) = D_{CS}(p_{X_{12}}, p_{X_1} p_{X_2}). \tag{7}$$

We now derive a closed-form solution for I_{CS} for Gaussian measurements of a discretized distribution, generalizing the result by Charrow [18] for ranging sensors. Consider a Gaussian measurement $\mathcal{Z}$ and a target random variable Θ with discretization $\hat{\Theta} \subset \Theta$, cell volume δ, and probability density $p_\theta = p(\hat{\Theta} = \theta)$, obtained directly for a histogram filter or by binning for a particle filter. Integrals over Θ of a function $\phi \colon \Theta \to \mathbb{R}$ are approximated as

$$\int_\Theta \phi(\theta) d\theta \approx \sum_{\theta \in \hat{\Theta}} \phi(\theta) \delta. \tag{8}$$

For convenience, denote (5) as $\mathcal{N}_\theta(z) = \mathcal{N}(z - \hat{f}_c^\theta, \sigma^2)$ using $\hat{f}_c^\theta$ to indicate dependence of (4) on θ. Substituting the joint, $p_{\theta z} = p_\theta \mathcal{N}_\theta(z)$ and product of marginals, $p_\theta p_z =$

$p_\theta \sum_{\theta' \in \hat\Theta} p_{\theta'} \mathcal{N}_{\theta'}(z)$ into (7), I_{CS} becomes

$$
\begin{aligned}
I_{CS}(\Theta; \mathcal{Z}) &\approx \log V_J + \log V_M - 2 \log V_C \\
V_J &= \sum_{\theta \in \hat\Theta} \int \mathcal{N}_\theta^2(z) p_\theta^2 dz \\
V_M &= \sum_{\theta \in \hat\Theta} \int p_\theta^2 \Bigg(\sum_{\theta' \in \hat\Theta} \mathcal{N}_{\theta'}(z) p_{\theta'} \Bigg)^2 dz \\
V_C &= \sum_{\theta \in \hat\Theta} \int \mathcal{N}_\theta(z) p_\theta^2 \sum_{\theta' \in \hat\Theta} \mathcal{N}_{\theta'}(z) p_{\theta'} dz
\end{aligned}
\tag{9}
$$

where V_J, V_M, and V_C are the joint, marginal and cross-potentials, using conventions by Príncipe [17] and omitting δ which cancels. The resulting closed-form solution is

$$
\begin{aligned}
&V_J = \sum_{\theta \in \hat\Theta} \mathcal{N}_{\theta\theta} p_\theta^2 \quad V_M = \sum_{\theta \in \hat\Theta} p_\theta^2 \sum_{\theta \in \hat\Theta} \sum_{\theta' \in \hat\Theta} \mathcal{N}_{\theta\theta'} p_\theta p_{\theta'} \\
&V_C = \sum_{\theta \in \hat\Theta} \sum_{\theta' \in \hat\Theta} \mathcal{N}_{\theta\theta'} p_\theta^2 p_{\theta'}
\end{aligned}
\tag{10}
$$

where $\mathcal{N}_{\theta\theta'}$ is the integral of the product of Gaussians

$$
\mathcal{N}_{\theta\theta'} = \int \mathcal{N}_\theta(z) \mathcal{N}_{\theta'}(z) dz = \mathcal{N}(\hat f_c^\theta - \hat f_c^{\theta'}, 2\sigma^2). \tag{11}
$$

This result is similar to those of Torkkola [19] and Príncipe [17]. Efficient approximation may be achieved using methods similar to those described by Seth and Príncipe [20].

E. Iterative estimation and deployment

Simultaneous deployment of the multi-robot team and estimation of mass properties is achieved by using informative measurements to characterize feasible lifting configurations via the previously described methods.

For problems consisting of only estimation, the mass properties can be estimated using greedy maximization of I_{CS}. Given choice of a measurement, $\mathcal{Z}_a$, at any attachment point $a \in \mathcal{A}$, a measurement is taken at

$$
\arg\max_{a \in \mathcal{A}} I_{CS}(\mathcal{Z}_a; \Theta) \tag{12}
$$

and is computed by evaluating I_{CS} at each attachment point.

We now employ cooperative measurements and extend (12) through heuristics to develop Alg. 1, an iterative algorithm for parameter estimation and formation of lifting configurations. The choice of whether to attach an additional robot is made based on the immediate increase in MI for measurements incorporating that robot. A new robot is attached only if doing so provides an increase in information gain greater than a factor of a parameter $\alpha \geq 1$ to encourage exploitation of already attached robots. We further introduce a chance constraint on the set of attachment points based on the probability that a feasible lifting configuration can be achieved to encourage formation of such configurations. Define the random variable $\mathcal{L}_\mathcal{R} \in \{0, 1\}$ where $\mathcal{L}_\mathcal{R} = 1$ if and only if there exists a feasible solution to (3) given $\mathcal{R}$. Then $\mathbb{E}_\Theta(\mathcal{L}_\mathcal{R})$ is the probability given Θ that a feasible lifting configuration can be formed by adding robots to $\mathcal{R}$. A new robot is only allowed to attach if this feasibility

Algorithm 1: Deployment cycle

```
A ← attachment points
R ← occupied attachment points, R ⊆ A
M ← argmax_{M ∈ C_{n_r}(R)} I_CS(Z_M; Θ)
a, M' ← argmax_{a ∈ Â_f, M' ∈ C_{n_r}(R ∪ {a})} I_CS(Z_M'; Θ)
if |R| < n_r and I_CS(Z_M'; Θ) > α I_CS(Z_M; Θ) then
    R ← R ∪ {a} (Attach new robot)
    Execute measurement Z_M'
else
    Execute measurement Z_M
```

is reduced by no more than a factor of γ^Δ where $0 < \gamma < 1$ and Δ is the number of iterations since a robot was last attached, inducing an exponential decay. The set of remaining attachment points with acceptable feasibility is $\hat{\mathcal{A}}_f \subseteq \hat{\mathcal{A}}$ defined as

$$
\hat{\mathcal{A}}_f = \{a | a \in \hat{\mathcal{A}}, \mathbb{E}_\Theta(\mathcal{L}_{\mathcal{R} \cup \{a\}}) \geq \gamma^\Delta \mathbb{E}_\Theta(\mathcal{L}_\mathcal{R})\}. \tag{13}
$$

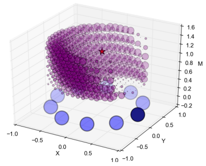
III. Results

The proposed approach is evaluated by first studying active estimation in isolation using (12) in order to demonstrate convergence rates. Further results consider deployment and then formation of lifting configurations, introducing Alg. 1 incrementally. The results address the case of $n_r = 4$ robots which provides flexibility over the minimum of 3 required for lifting. Although the approach extends to any number of robots, the combinatorial nature of the set of cooperative measurements makes this impractical. Achieving scalability using numerical optimization of cooperative measurements is left to future work.

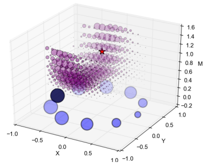
The object to be manipulated is a cylinder with radius 1 m with contacts approximated by 20 points on the boundary. The histogram filter has discretization $\hat\Theta$ with a spatial resolution of 0.1 m and mass resolution of 0.2 kg ranging from 0.6 kg to 1.4 kg. The CoM and mass are sampled uniformly from the support unless otherwise specified. There are 10 attachment points, $|\mathcal{A}| = 10$ distributed uniformly on a concentric circle with radius 0.8 m. Measurements have uncertainty $\sigma = 1$ N m which is injected into the system and considered in Bayesian updates.

Parameter estimates are computed by a weighted average over the belief distribution of the histogram filter. The Euclidean norm of normalized parameter error is used with each parameter scaled by the reciprocal of its range (2 m, 2 m, and 0.8 kg respectively) because of differing units and for concise presentation. Due to the resolution of the filter, a maximum accuracy of approximately 0.1 normalized units is achieved on average.

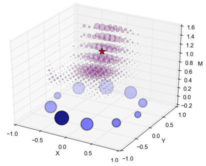
Results comparing greedy (12) to uniform random and deterministic cyclic action selection are shown in Fig. 4. The cyclic approach selects measurements at uniform rates and avoids taking nearby measurements by incrementing the



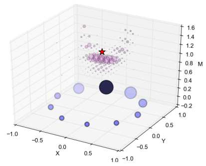
(a) Iteration 1



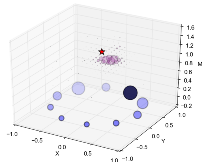
(b) Iteration 2



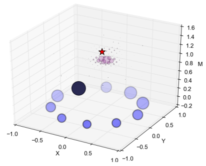
(c) Iteration 3



(d) Iteration 4



(e) Iteration 5



(f) Iteration 6

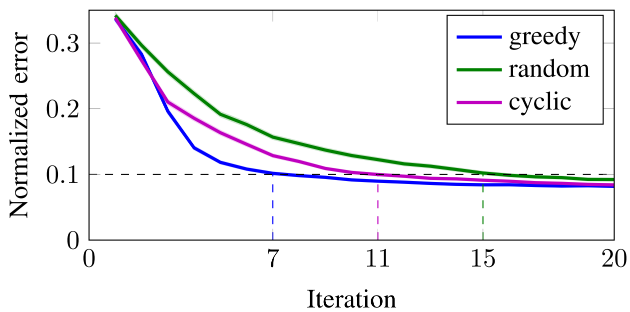


Fig. 4: *Pure estimation*: (top) An example of greedy action selection with actions (denoted as blue circles) scaled by I_{CS}, and the maximizer is darkened. Belief (denoted by purple circles) is shown following execution of the chosen action with the true CoM and mass denoted as a red star. (bottom) Normalized error for greedy, random, and cyclic selection along with the standard error. The greedy algorithm converges more than twice as quickly to resolution limits as random selection and fifty-percent faster than cyclic selection as indicated by dashed lines.

index of the attachment point by three after each iteration. The test consists of 1000 trials with 20 iterations per trial and does not consider actuator constraints. Included examples show the evolution of the belief distribution and the subsequent variation of I_{CS}. Although each approach converges, greedy action selection converges more than twice as fast as random selection and more than fifty-percent faster than cyclic selection, reaching a mean accuracy of 0.1 after 7 iterations while random and cyclic selection require 15 and 11 iterations respectively to reach the same accuracy. Even in this simple case, maximizing MI significantly outperforms reasonable uninformed approaches. The remaining scenarios address harder problems with larger spaces of actions and measurements that may not be discriminative due to actuator constraints.

Having demonstrated use of I_{CS} as a cost function, consider the problem of lifting and deployment. Actuator constraints are now enforced with a limit of $f_{\max} = 4$ N per robot for an *estimation and deployment* task and $f_{\max} = 6$ N for *feasible lifting*. Given the maximum object weight of $f_g = 13.7$ N, measurements frequently reach actuator limits in the estimation task with a mean rate of 1.45 times per trial and maximum of 13 times over all trials, during the lifting

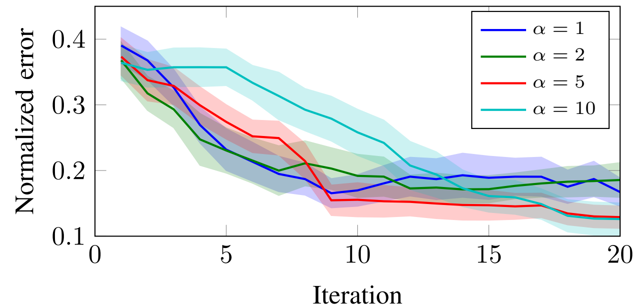


Fig. 5: Normalized error for the *estimation and deployment* task with $f_{\max} = 4$ N and various values of α. In this scenario, robots iteratively attach to the object (following Fig. 3) and estimate its mass properties according to Alg. 1. Groups of attached robots execute cooperative measurements in order to estimate the properties of objects with mass that may exceed the capabilities of individual robots.

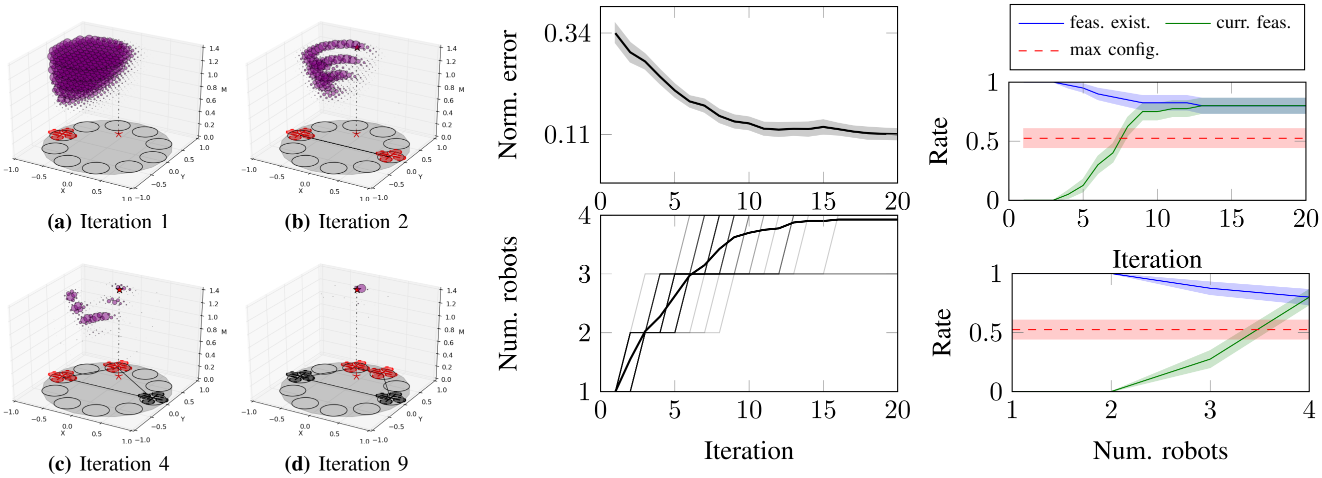
task, four robots are generally necessary before being able to lift the object.

For *estimation and deployment*, Fig. 5 shows results for various values of α over 20 trials per each value. In this test, Alg. 1 is modified slightly, discarding the chance constraint and choosing robots from $\hat{\mathcal{A}}$ as the goal is estimation rather than lifting. Although not all differences are significant, $\alpha = 5$ achieves a balance between initial convergence and final error. In general, estimates now converge more slowly than for *pure estimation* due to the additional constraints.

Figure 6 shows results of 40 trials of the full *feasible lifting* task using $\alpha = 5$ and $\gamma = 0.9$. Parameters, θ, are now sampled from the subset having feasible lifting configurations shown in Fig. 3a (to emphasize discovery of feasible configurations). Final error remains comparable to previous experiments with a mean of 0.11 normalized units. To form a baseline for performance in formation of feasible lifting configurations, we compare to the best individual configuration over the space of parameters (shown in Fig. 3b) which has an expected success rate of 47.6% given that a feasible configuration exists. The approach achieves a feasible lifting configuration in the sense of (2) in 80% of trials which is a significant improvement compared to 53.5% for the baseline configuration. In all except three trials all four robots attach, and in each of these cases the three attached robots achieve a feasible lifting configuration.

IV. Conclusion and Future Work

This work proposes an approach to enable teams of aerial robots to lift unknown objects using Bayesian filtering to estimate object parameters and active sensing to select informative interactions coupled with a chance constrained deployment strategy to form feasible lifting configurations. Existing approaches to aerial manipulation require assumptions on robot configurations and object mass. The proposed approach relaxes assumptions that mass properties are known and proposes a novel approach to estimation that permits safe



(a) Iteration 1 **(b)** Iteration 2 **(c)** Iteration 4 **(d)** Iteration 9

Fig. 6: *Feasible lifting*: (left) An example of one trial showing attachment points $\mathcal{A}$ (black circles), the measurement set $\mathcal{M}$ (red), and remaining attached robots $\mathcal{R} \setminus \mathcal{M}$ (black) with the belief shown as in Fig. 4. (center-top) Normalized error and standard error. (center-bottom) Number of robots attached with mean behavior (black) and individual trials (gray). (right) Fraction of trials for which a feasible configuration exists (blue) by (3), has been achieved (green) by (2), and success rate (red) of the baseline (Fig. 3b) each by iteration and number of robots. Trials that did not reach a given number of robots are all successful and treated as feasible for addition of a fourth robot.

estimation of the mass and CoM through wrenches applied to the object. An information-theoretic objective is derived to permit active selection of maximally informative measurements. The results demonstrate both rapid convergence in estimation and successful formation of feasible lifting configurations in 80% of trials.

A number of simplifications are made in the scenarios investigated in the results. The measurements are formulated as a continuous set of rays in wrench-space, but the results consider a finite subset of possible measurements. Numerical optimization techniques are likely to provide benefits both in computation and estimation. Greedy strategies are used for planning to emphasize contributions in estimation and active sensing. Better planning approaches can be developed. A finite horizon planner would be especially beneficial in formation of lifting configurations while the proposed approach allows for evaluation of I_{CS} over sets of measurements occurring in such planning approaches. Experimental evaluation will enable further evaluation of the approach, and implementation within the context of general aerial manipulation is desirable.

References

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